Introduction.

In his paper L6, Langlands shows how the zeta function of certain Shimura varieties can be expressed as a product of L-functions associated to automorphic representations of the algebraic group G entering the description of the Shimura variety (or rather, the endoscopic groups for G). The group G is here (roughly speaking) obtained by scalar reduction to $\mathbb{Q}$ of the multiplicative group of a certain quaternion algebra over an algebraic numberfield. The paper L6 is concerned with the local zeta function of the variety obtained by reducing the Shimura variety at a (finite) place of its definition field where it has good reduction, and it is based on a description of this reduced variety which was unproven (and which was formerly presented in L2 and L3 - a more detailled account can be found in M1 and M2).

L6 is a contribution to a theory which in some future should tell us how we can generalize some classical results, such as that (due to Eichler) saying that the zeta function of a modular curve $\Gamma\backslash H$ (H the upper halfplane and Γ some congruence subgroup of $SL_2(\mathbb{Z})$) can be expressed as a product of L-functions associated to automorphic forms on $\Gamma\backslash H$ (or otherwise speaking, to automorphic representations of $GL_2(\mathbb{A})$), can be analytically continued and that the analytic continuation satisfies a functional equation.

The proofs of the classical results are based on congruence relations between Hecke operators and the Frobenius, and this method does not seem to work for general Shimura varieties.

The proof in L6 is based on the Selberg trace formula and is in some simpler cases presented in L3, Ca and La (see also BL, HLR and Ra), but the cases studied in L6 take care of a complication that arises by the fact that whereas a L-function is associated not to a single representation but to an L-indistinguishable class of representations of $G(\mathbb{A})$, two L-indistinguishable representations can occur with different multiplicity in $L^2(G(\mathbb{Q})Z(\mathbb{R})\backslash G(\mathbb{A}))$. This misfortune can be restored by using L-functions not associated to representations of G, but to representations of the so-called endoscopic groups for G. Even though the endoscopic groups in the cases studied in L6 are of a rather simple type, as they are either elliptic Cartan subgroups of G or the quasi-split inner form of G, L6 gives nevertheless a clear picture of the way in which they come into play in the general case.

Two circumstances however make it difficult immediately to generalize the method of L6. A class decomposition of the points of the reduced variety is parametrized by equvivalence classes of so-called Frobenius pairs, but different domains can correspond to the same equvivalence class because the equvivalence relation is of local nature where it ought to be of global nature. Moreover the number of points left fixed by a power of the Frobenius is calculated explicitely by a complicated combinatoric argument.

In Langlands and Rapoport's paper LR the first difficulty is remedied - the description of the points conjectured there is more elegant and will possibly cover also the case of bad reduction (see Ra), and (expect for some standard conjectures of algebraic geometry) it is proved to be true in the case of good reduction for certain Shimura varieties that parametrize families of polarized abelian varieties with endomorphism- and level structure.

In Kottwitz's paper K4 a special case is worked out of an idea which seems to make it possible to reduce all the combinatoric calculations in L6 to some standard problems in harmonic analysis: the relation between orbital- resp. twisted orbital integrals of associated functions in the case of passing to endoscopic groups resp. the case of base change.

In the present paper I will show - by using primary the material of LR and K4, and, of course, building on the ideas and techniques of L6 - how a proof for the expression of the "tempered cuspidal" part of the local zeta function in terms of L-functions in the case of a general Shimura variety should be set up: the proof will built on some precisely formulated conjectures of general nature. The purely formal part of the proof is presented in section 2, section 1 is devoted an explanation of each step of section 2, and section 3 is a list of all conjectures used.

It is necessarily to presuppose that the reductive $\mathbb{Q}$-group G is such that G_{der} is simply connected - why and how the general case can simply be reduced to this case is explained in LR. Moreover the Shimura variety in question is assumed to be of compact type, that is, its points with coordinates in $\mathbb{C}$ is a compact space, this amounts to demand that G_{ad} is anisotropic over $\mathbb{Q}$.